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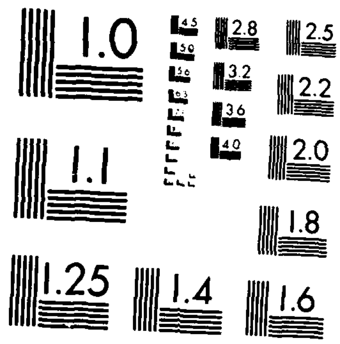

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New developments in computer architecture (parallel and multi-processors) and a renewed interest in 3 dimensional problems and higher-order problems have revived interest in iterative methods for elliptic and parabolic finite-difference/finite/element equations.

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An Overview

New developments in computer architecture (parallel computers and multi-processors) and a renewed interest in 3-dimensional problems and higher-order problems have revived interest in iterative methods for elliptic and parabolic finite-difference/finite-element equations.

Consider the linear system

$$(1) \qquad Ax = y$$

which arises from the discretization of an elliptic boundary-value problem. A direct (linear stationary) iterative method arises from the splitting

$$(2) \qquad A = M - N$$

and takes the form

$$(3) \qquad Mx^{k+1} = Nx^{k} + y \; .$$

The study of the rates of convergence of such an iterative scheme is essentially the study of the spectral radius ρ of the iteration matrix $K = M^{-1}N$.

Parter has been studying these problems for over 20 years. Since 1977 he has been working with several researchers at the Los Alamos National Laboratory, studying methods which are easily implemented on the CRAY. This work has led to several reports and publications. Most of this work has been concerned with finite-difference equations. However, in [1] Parter and Steuerwalt developed the theory for finite-element equations. In a certain sense this paper was the culmination of this line of research. The survey papers [2] and [7] describe the general approach, the basic results and the significance of these results in applications.

This theory yields results of the form

$$(4) \qquad \rho = 1 - Ch^p + o(h^p)$$

$$(5) \qquad 1 > \rho \geq 1 - Ch^p + o(h^p)$$

where $C > 0$, $p > 0$ are constants determined by the problem, and h is the discretization parameter which is going to zero. Unfortunately, in any practical situation, h is small and ρ is near "1". Thus, while the results are useful and properly describe the rates of convergence of many of the methods currently in use, the results also imply that it is imperative that we consider other solution schemes.

In recent years there have been two new approaches to this problem: (1) Multigrid methods and (2) (wisely) preconditioned conjugate gradient methods. In both of these approaches, which are intrinsically related, one obtains error estimates of the form

$$\|\varepsilon^{(k+1)}\|_A \leq \rho\|\varepsilon^{(k)}\|_A \tag{6}$$

where ε^k and ε^{k+1} are the errors at the k^{th} and (k+1)st iteration, $\| \quad \|_A$ denotes the operator norm and ρ is a constant, independent of h, $0 < \rho < 1$.

However it seems we are just beginning to understand these powerful methods. In particular, there are the questions: (1) in preconditioning methods, how does one choose an *effective* preconditioner, (2) in multigrid methods, how do we choose the interpolation and projection operators? The smoothing operators? And, how can we obtain sharp, numerical estimates on the quantity ρ ?

Working with David Kamowitz, a graduate student in the Computer Sciences Department who has just completed his phd., Parter has been involved in several multigrid projects on both a theoretical and an experimental level.

In [3] they studied difference equations for boundary value problems for ordinary differential equations of the form

$$-(pu')' + bu' + qu = f\,, \quad 0 \leq x \leq 1 \tag{7a}$$

$$u(0) = u(1) = 0\,. \tag{7b}$$

This paper contains both a theoretical multigrid analysis and a computational study. In [13] Kamowitz extends these ideas to study singular perturbation problems with and without turning points. Thus, [13] is concerned with problems of the form

$$-\varepsilon u'' + bu' = f\,, \quad 0 \leq x \leq 1 \tag{8a}$$

$$u(0) = u(1) = 0 \,. \tag{8b}$$

In [4] and [5] Kamowitz and Parter discuss the $MGR[\nu]$ methods for the general diffusion equation

$$\nabla \cdot a(x,y) \nabla u = f, \quad \Omega \,, \tag{9a}$$

$$u = 0, \quad \partial\Omega \,. \tag{9b}$$

Earlier studies were limited to the Poisson equation.

In [8] Parter gives an analysis of the three-grid scheme for equations (9a), (9b). In [6] and [10] Parter discusses various aspects of the state of the convergence theory for multigrid methods. In [11] Kamowitz describes a parallel implementation of $MGR[\nu]$ methods and SOR methods making use of the UW CRYSTAL project. In [14] Kamowitz describes an experimental multigrid study for the solution of the steady state neutron diffusion equation

$$\psi(t) = \frac{\gamma}{2} \int_0^1 E_1(|z - z'|)\psi(z')dz' + h(z) \,, \quad 0 \le z \le 1 \,. \tag{10}$$

The lengthy report [12] by Faber, Manteuffel and Parter developes some basic ideas of "norm equivalence" as well as establishing some new results on effective preconditioning methods for the important "indefinite case". Earlier studies of preconditioning methods have been based on the concept of "spectral equivalence". Unfortunately, this approach limits one to the case where the symmetric part of the operator is positive definite.

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PUBLICATIONS

1. Parter, S. V. and M. Steuerwalt, Block Iterative methods for elliptic finite-element equations, SIAM J. Numer. Anal., *22* (1985), 146-179.

2. Parter, S. V. and M. Steuerwalt, Iterative Methods for Discrete Elliptic Equations. Proc. Sixth International Conference on Computing Methods in Engineering and Applied Science, 47-56, Versailles, France - December 12-16, 1983, Edited by R. Glowinski and J. L. Lions, North Holland Publishers, Amsterdam (1981).

3. Kamowitz, D. and S. V. Parter, A study of some multigrid ideas, Appl. Math. Computation, *17* (1985), 153-184.

4. Kamowitz, D. and S. V. Parter, Multigrid and MGR[ν] methods for diffusion equations. Full paper appears in Journal of Computational Math *3* (1985) 373-384 (Chinese Journal). Extended abstract appears in Proceedings DD5 Conference on Numerical Methods for Partial Differential Equations, Beijing, China, August 13-17, 1984. Edited by Feng Kang, Science Press, Beijing, China (1985).

5. Kamowitz, D. and S. V. Parter, On MGR[ν] multigrid methods, University of Wisconsin-Madison, Computer Sciences Department Technical Report #575 (1985). *To appear*: SIAM J. Numer. Anal.

6. Parter, S. V., A note on convergence of the multigrid V-cycle, Appl. Math. Comput., *17* (1985), 137-151.

7. Parter, S. V., Iterative methods for elliptic problems and the discovery of "q", SIAM Rev., *28*, (1986) 153-175.

8. Parter, S. V., On an estimate for the three-grid MGR multigrid method, University of Wisconsin-Madison, Computer Sciences Department Technical Report #610, (1985).

9. Parter, S. V., On the distribution of the singular values of Toeplitz matrices. University of Wisconsin-Madison, Computer Sciences Department Technical Report #609 (1985). *To Appear*: J. Lin. Alg. and Appl.

10. Parter, S. V., Remarks on Multigrid Convergence Theorems, University of Wisconsin - Madison, Computer Sciences Department Technical Report #634, (1986).

11. Kamowitz, D., SOR and MGR[ν] experiments on the CRYSTAL and multicomputer. University of Wisconsin-Madison, Computer Sciences Department Technical Report #623. *To Appear*: Parallel Computing.

12. Faber, V., T. A. Manteuffel and S. V. Parter, On the equivalence of operators and the implications to preconditioned iterative methods for elliptic problems. Los Alamos National Laboratory Report #LA-UR-86-2152 (1986).

13. Kamowitz, D., Further results on one-dimensional multigrid. In preparation.

14. Kamowitz, D., An experimental study of the application of multigrid to the integral equation of neutron transport. In preparation.

Contacts with other Research Groups

1. Professor Parter is a member of the SCIENCE COUNCIL FOR ICASE (Institute for Computer Applications in Science and Engineering, NASA Langley Research Center, Hampton, Virginia). In this capacity he visits ICASE one or two times each year. During these visits he spends some time discussing scientific matters with the staff.
2. Professor Parter is a member of the Panel for Applied Mathematics for the Center for Applied Mathematics at the National Bureau of Standards. In this capacity he visits the Center for Applied Mathematics at least once a year. During these visits he spends some time discussing scientific matters with the staff.
3. Professor Parter is a regular visitor to the Los Alamos National Laboratory where he collaborates with the staff of the numerical analysis group C-3. Some of this work is closely related to the work under this contract.

Lectures and Visits

I. The following is a partial list of Professor Parter's lectures and visits.

1. NASIG (Numerical Analysis Special Interest Group of the National Laboratories) meeting, June 23-25, 1982, Los Alamos Scientific Laboratory. Lecture : Block Iterative Methods for Finite Element Elliptic Equations.
2. International Multigrid Conference, April 6-8, 1983, Copper Mountain, Colorado.
3. Conference on Large Scale Scientific Computation, May 17-19, 1983, Madison, WI. Professor Parter was the chairman of the organizing committee and the editor of the published proceedings.
4. First Army Conference on Applied Mathematics and Computing, May 9-11, 1983. George Washington University, Washington, D.C. Lecture : On the Solution of Elliptic Finite-Element Equations.
5. National SIAM (Society for Industrial and Applied Mathematics) October 1983. Lecture : Iterative Methods for Elliptic Problems and the Discovery of "q". Retiring Presidential Address.
6. Sixth International Conference on Computing Methods in Applied Sciences and Engineering, December 12-16, 1983 Versailles, France. Lecture : Iterative Methods for Discrete Elliptic Equations. Note : all lectures at this conference are by invitation of the organizing committee.
7. Visit to Tel-Aviv University, December 17, 1983-Jan. 10, 1984. Colloquium Lecture : Some Multigrid Ideas.
8. DD5 Conference on Numerical Methods for Partial Differential Equations, August 13-17, 1984. Beijing, China. Invited Lecture : Multigrid and MGR[ν] Methods for Diffusion Problems.
9. Workshop on Computational Mechanics, September 24-25, 1984. NASA Lewis Research Center, Cleveland, Ohio. Professor Parter was the chairman of the organizing committee.
10. Second Copper Mountain Conference on Multigrid Methods, March 31-April 3, 1985.

11. Second SIAM Conference on Linear Algebra, April 28-May 1, 1985, Raliegh, North Carolina. Invited Lecture : Spectral Estimates for families of matrices.
12. Visit to Southern Methodist University, September 18-19, 1985. Colloquium Lecture : Multigrid Estimates.
13. Visit to Brookhaven National Laboratory, February 12, 1986. Discussions with Charles Goldstein and others.
14. Visit to Brown University, Department of Applied Mathematics, February 13-14, 1986. Colloquium Lecture : Multigrid.
15. Visit to University of Colorado in Denver, April 9, 1986. Colloquium Lecture : Multigrid Convergence Theorems.
16. Visit to New York University - Courant Institute for Mathematical Sciences, May 12, 1986. Discussions with Peter Lax, Jonathan Goodman and others.

II. The following is a list of David Kamowitz's lectures and visits.

1. Second Copper Mountain Conference on Multigrid Methods, March 31-April 3, 1985. Lecture : Some CRYSTAL Experiments on Multigrid.
2. ICASE, NASA Langley Research Center, Hampton Virginia, February 20, 1986. Lecture : MGR multigrid - CRYSTAL experiments.
3. IBM Research Center, Yorktown Heights, New York, February 24, 1986. Lecture : MGR multigrid - CRYSTAL experiments.
4. Computer Science Department, University of Colorado - Boulder. March 3, 1986. Lecture : MGR multigrid - CRYSTAL experiments.
5. Bell Telephone Laboratories, Murray Hill, N.J. March 11, 1986. Lecture : MGR multigrid - CRYSTAL experiments.
6. Argonne National Laboratory, Applied Mathematics Department. March 17, 1986. Lecture : MGR multigrid - CRYSTAL experiments.

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